

Algebraic Solutions for $U^{BF}(5) - O^{BF}(6)$ Quantum Phase Transition in Odd Mass Number Nuclei

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The spherical to gamma-unstable nuclei shape- phase transition in odd-A nuclei is investigated by using the Dual algebraic structures and the affine $\widehat{SU}(1,1)$ Lie Algebra within the framework of the interacting boson - fermion model. The new algebraic solution for A-odd nuclei is introduced. In this model, Single $j = 1/2$ and $3/2$ fermions are coupled with an even-even boson core. Energy spectra, quadrupole electromagnetic transitions and an expectation value of the d-boson number operator are presented. Experimental evidence for the $U^{BF}(5) - O^{BF}(6)$ transition in odd -A *Ba* and *Rh* isotopes is presented. The low-states energy spectra and $B(E2)$ values for these nuclei have been also calculated and compared with the experimental data.

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I. INTRODUCTION

Quantum Phase Transitions ($QPTs$) are sudden changes in the structure of a physical system. Nuclear physics has important contributions to make to their study because nuclei display a variety of phases in systems ranging from few to many particles [1]. The signs of $QPTs$ in nuclear physics are changes in mass and radius of nuclei and quantities such as level crossing and electromagnetic transition rates when the number of protons or neutrons is modified. Phase transition happens in both even-even and *odd* - A nuclei. Phase transitions investigations have been mostly performed on even-even systems [2] within the framework of the interacting boson model (IBM) [2, 3], which describe nuclei in terms of correlated pairs of nucleons with $L = 0, 2$ treated as bosons (s, d bosons). The IBM Hamiltonian has exact solutions in three dynamical symmetry limits ($U(5), SU(3)$, and $O(6)$). These situations correspond to the spherical, axially deformed, and gamma-unstable ground state shapes, respectively. The transition between the $U(5)$ - $SU(3)$ limits is a first-order shape-phase transition while a second-order shape-phase transition occurs between the $U(5)$ and $O(6)$ limits [4–6]. During a transition from one limit to another, meet the points in which potential has flat behavior. These points are called critical point. Recently Iachello introduced the so-called critical point symmetries in the framework of the collective model for even-even nuclei. The critical point from spherical to γ - *unstable* shapes, called $E(5)$ [3, 6], the critical point from spherical to axially deformed shapes, called $X(5)$ [4, 6], and the critical point from axially deformed shapes to triaxial shapes, called $Y(5)$ [6, 7]. Phase transitions is also investigate in odd- A nuclei within the framework of interacting boson - fermion model ($IBFM$) [8], which describe nuclei in terms of correlated pairs, with $L = 0, 2$ (s, d bosons), and unpaired particles of angular momentum j (j fermions). Studies of $QPTs$ in odd-even nuclei were implicitly initiated years ago by Scholten and Blasi [9]. Several explicit studies have recently been made by Alonso et al. [10–12] and by Boyukata et al. [13], who also have suggested a simple form of the $IBFM$ Hamiltonian, particularly well-suited to study $QPTs$ in odd-even nuclei because of their supersymmetric properties. Similar to even-even nuclei, also exists critical point for odd-even and even-odd nuclei but in this case, critical points show with $E(5/\sum_j 2j + 1)$ and $X(5/\sum_j 2j + 1)$ that j is the angular momentum of single nucleon. Iachello [14, 15] has been the case of a $j(3/2)$ fermion coupled to a boson core that undergoes a transition from spherical to γ - *unstable* shapes. At the critical point, an elegant analytic solution, called $E(5/4)$, has been obtained starting from the Bohr Hamiltonian [14].

In this study, we investigate the transition $U^{BF}(5) - O^{BF}(6)$ in *odd* - A nuclei. The new algebraic solution for A -odd nuclei is introduced. For this transition only the boson core experience the transition and fermion with $j = 1/2$ and $3/2$ coupled to boson core. We evaluate exact solutions for eigenstate and energy eigenvalues for transitional region in the $IBFM$ by using the Dual algebraic structure for the two level pairing model that based on Richardson - Gaudin method and changing the control parameter that based on affine $SU(1, 1)$ lie algebra. In order to the investigation of phase transition, we calculate observables such as level crossing, expectation values of the d -boson number operator, ground - state energy and its first derivative. The low-lying states of $^{127-137}_{56}Ba$ and $^{101-109}_{45}Rh$ isotopes have been studied within suggested model. The results of calculations for these nuclei will present for energy levels and transitions probabilities, two neutron separation energies and will compare with the corresponding the experimental data.

This paper is organized as follows: Section 2 briefly summarizes theoretical aspects of transitional Hamiltonian and affine $SU(1, 1)$ algebraic technique. Sections 3 and 4 include the results and Experimental evidence and sect. 5 is devoted to the summary and some conclusions.

II. THEORETICAL FRAMEWORK

The $SU(1, 1)$ Algebra has been explained in detail in Refs [16–18]. The $SU(1, 1)$ algebra is produced by S^ν , $\nu = 0$ and \pm , which satisfies the following commutation relations

$$[S^0, S^\pm] = \pm S^\pm, \quad [S^+, S^-] = -2S^0 \quad (1)$$

The quadratic Casimir operator of $SU(1, 1)$ can be written as

$$\hat{C}_2 = S^0(S^0 - 1) - S^+ S^- \quad (2)$$

The basis states of an irreducible representation (*irrep*) $SU(1, 1)$, $|k\mu\rangle$, are determined by a single number k , k where can be any positive number and $\mu = k, k + 1, \dots$. Therefore [17, 18],

$$\hat{C}_2(SU(1, 1))|k\mu\rangle = k(k - 1)|k\mu\rangle, \quad S^0|k\mu\rangle = \mu|k\mu\rangle \quad (3)$$

In IBM , the generators of $SU^d(1,1)$ generated by the d-boson pairing algebra

$$S^+(d) = \frac{1}{2}(d^+ \cdot d^+) \quad , \quad S^-(d) = \frac{1}{2}(\tilde{d} \cdot \tilde{d}), \quad S^0(d) = \frac{1}{4} \sum_{\nu} (d_{\nu}^+ d_{\nu} + d_{\nu} d_{\nu}^+) = \frac{1}{4}(2\hat{n}_d + 5) \quad (4)$$

Similarly, s- boson pairing algebra forms another $SU^s(1,1)$ algebra generated by

$$S^+(s) = \frac{1}{2}s^{+2} \quad , \quad S^-(s) = \frac{1}{2}s^2, \quad S^0(s) = \frac{1}{4}(s^+ s + s s^+) = \frac{1}{4}(2\hat{n}_s + 1) \quad (5)$$

$SU^{sd}(1,1)$ is the s and d boson pairing algebras generated by

$$S^+(sd) = \frac{1}{2}(d^+ \cdot d^+ \pm s^{+2}) \quad , \quad S^-(sd) = \frac{1}{2}(\tilde{d} \cdot \tilde{d} \pm s^2) \quad , \quad S^0(sd) = \frac{1}{4} \sum_{\nu} (d_{\nu}^+ d_{\nu} + d_{\nu} d_{\nu}^+) + \frac{1}{4}(s^+ s + s s^+) \quad (6)$$

Because of duality relationships [19, 20], It is known that the base of $U(5) \supset SO(5)$ and $SO(6) \supset SO(5)$ are simultaneously the basis of $SU^d(1,1) \supset U(1)$ and $SU^{sd}(1,1) \supset U(1)$, respectively. By use of duality relations [17, 19], the Casimir operators of $SO(5)$ and $SO(6)$ can also be expressed in terms of the Casimir operators of $SU^d(1,1)$ and $SU^{sd}(1,1)$, respectively

$$\hat{C}_2(SU^d(1,1)) = \frac{5}{16} + \frac{1}{4}\hat{C}_2(SO(5)) \quad (7)$$

$$\hat{C}_2(SU^{sd}(1,1)) = \frac{3}{4} + \frac{1}{4}\hat{C}_2(SO(6)) \quad (8)$$

The infinite dimensional $SU(1,1)$ algebra that is generated by use of [17, 18]

$$S_n^{\pm} = c_s^{2n+1} S^{\pm}(s) + c_d^{2n+1} S^{\pm}(d) \quad , \quad S_n^0 = c_s^{2n} S^0(s) + c_d^{2n} S^0(d) \quad (9)$$

Where c_s and c_d are real parameters and n can be $0, \pm 1, \pm 2, \dots$. These generators satisfy the commutation relations

$$[S_m^0, S_n^{\pm}] = \pm S_{m+n}^{\pm} \quad , \quad [S_m^+, S_n^-] = -2S_{m+n+1}^0 \quad (10)$$

Then, $S_m^{\mu}, \mu = 0, +, -; m = \pm 1, \pm 2, \dots$ generate an affine Lie algebra $\widehat{SU(1,1)}$ without central extension.

In odd A nuclei the Bose – Fermi symmetries are associated with each of the dynamic symmetries of IBM-1[8]. So, the boson algebraic structure will be always taken to be $U^B(6)$, while the fermion algebraic structure will depend on the values of the angular momenta, j , taken into consideration [8]. First we considered the case that a system of N bosons (with $L=0, 2$) coupled to a fermion with angular momentum $j=1/2$. The Lattice of algebras in this case is shown in Fig.1. In Figs 1, 2, the chain 1 show the state that bosons have $U^B(5)$ dynamical symmetric while bosons in chain 2 have $O^B(6)$ dynamical symmetric. By employing the generators of Algebra $\widehat{SU(1,1)}$ and Casimir operators of subalgebras, the following Hamiltonian for transitional region between $U^{BF}(5) - O^{BF}(6)$ limits is prepared

$$\hat{H} = gS_0^+ S_0^- + \alpha S_1^0 + \beta \hat{C}_2(SO^B(5)) + \delta \hat{C}_2(SO^B(3)) + \gamma \hat{C}_2(\text{spin}^{BF}(3)) \quad (11)$$

Following this, we considered the state that the odd nucleon being in a $j=3/2$ shell. The Lattice of algebras in this case is also shown in Fig.2. The following Hamiltonian for state that odd nucleon being in a $j=3/2$ shell for transitional region between $U^{BF}(5) - O^{BF}(6)$ limits is prepared

$$\hat{H} = gS_0^+ S_0^- + \alpha S_1^0 + \beta \hat{C}_2(\text{spin}^{BF}(5)) + \gamma \hat{C}_2(\text{spin}^{BF}(3)) \quad (12)$$

Eqs.(11) and (12) are the suggested Hamiltonians for boson - fermion systems with $j = 1/2, 3/2$, respectively and $\alpha, \beta, \delta, \gamma$ are real parameters. By considering Eqs.(2),(7),(8), It can be shown that Hamiltonians (11) and (12) are equivalent with $O^{BF}(6)$ Hamiltonian when $c_s = c_d$ and with $U^{BF}(5)$ Hamiltonian if $c_s = 0$ and $c_d \neq 0$. Thus, As mentioned because that only the boson core experience the transition and fermion coupled to boson core, the $c_s \neq c_d \neq 0$ situation just corresponds to $U^{BF}(5) \leftrightarrow O^{BF}(6)$ transitional region. In our calculation, we take $c_d (= 1)$

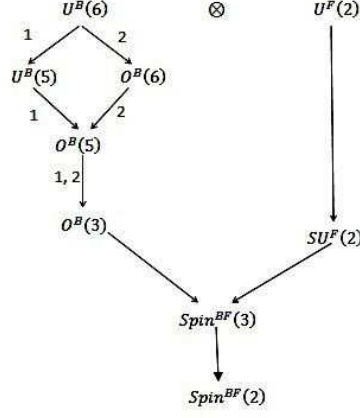


FIG. 1. The Lattice of algebras in the case that a system of N bosons (with $L = 0, 2$) coupled to a fermion with angular momentum $j = 1/2$.

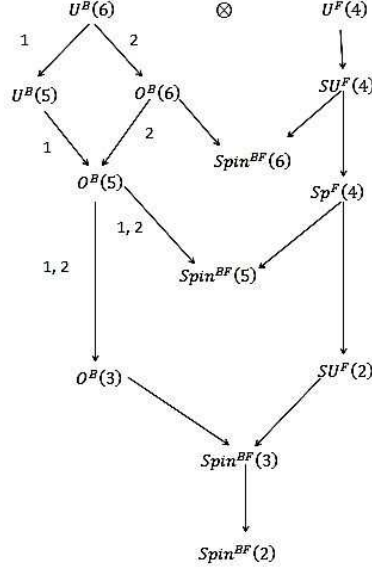


FIG. 2. The Lattice of algebras in the case that a system of N bosons (with $L = 0, 2$) coupled to a fermion with angular momentum $j = 3/2$.

constant value and c_s change between 0 and c_d . For evaluating the eigenvalues of Hamiltonians (11) and (12) the eigenstates are considered as [17, 18]

$$|k; \nu_s \nu n_\Delta LM\rangle = \sum_{n_i \in \mathbb{Z}} a_{n_1 n_2 \dots n_k} x_1^{n_1} x_2^{n_2} x_3^{n_3} \dots x_k^{n_k} S_{n_1}^+ S_{n_2}^+ S_{n_3}^+ \dots S_{n_k}^+ |lw\rangle \quad (13)$$

Eigenstates of Hamiltonians (11) and (12) can obtain with using the Fourier-Laurent expansion of eigenstates and $SU(1, 1)$ generators in terms of c-unknown number parameters x_i with $i = 1, 2, \dots, k$. It means, one can consider the

eigenstates as [17, 18]

$$|k; \nu_s \nu n_\Delta LM\rangle = N S_{x_1}^+ S_{x_2}^+ S_{x_3}^+ \dots S_{x_k}^+ |lw\rangle^{BF} \quad (14)$$

Where N is the normalization factor and

$$S_{x_i}^+ = \frac{c_s}{1 - c_s^2 x_i} S^+(s) + \frac{c_d}{1 - c_d^2 x_i} S^+(d) \quad (15)$$

The c-numbers x_i are determined through the following set of equations:

$$\frac{\alpha}{x_i} = \frac{g c_s^2 (\nu_s + \frac{1}{2})}{1 - c_s^2 x_i} + \frac{g c_d^2 (\nu_d + \frac{5}{2})}{1 - c_d^2 x_i} - \sum_{j \neq i} \frac{2g}{x_i - x_j} \quad (16)$$

With *Clebsch – Gordan* (CG) coefficient, we can calculate lowest weight state, $|lw\rangle^{BF}$, in terms of boson and fermion part. For the $j=1/2$ case we have:

$$|lw\rangle_{m \pm \frac{1}{2}}^B = |N, k_d = \frac{1}{2}(\nu_d + \frac{5}{2}), \mu_d = \frac{1}{2}(n_d + \frac{5}{2}), k_s = \frac{1}{2}(\nu_s + \frac{1}{2}), \mu_s = \frac{1}{2}(n_s + \frac{1}{2}), L, m \pm \frac{1}{2}\rangle \quad (17)$$

$$|lw\rangle^{BF} = \pm \sqrt{\frac{L \pm m + \frac{1}{2}}{(2L+1)}} |lw\rangle_{m - \frac{1}{2}}^B \chi_+ + \sqrt{\frac{L \mp m + \frac{1}{2}}{(2L+1)}} |lw\rangle_{m + \frac{1}{2}}^B \chi_- \quad (18)$$

The lowest weight state for the $j=3/2$ case is calculated as:

$$|lw\rangle_{m \pm \frac{3}{2}}^B = |N, k_d = \frac{1}{2}(\nu_d + \frac{5}{2}), \mu_d = \frac{1}{2}(n_d + \frac{5}{2}), k_s = \frac{1}{2}(\nu_s + \frac{1}{2}), \mu_s = \frac{1}{2}(n_s + \frac{1}{2}), L, m \pm \frac{3}{2}\rangle \quad (19)$$

$$\begin{aligned} |lw\rangle^{BF} = & C_{m, m - \frac{3}{2}, \frac{3}{2}}^{J, L, \frac{3}{2}} |lw\rangle_{m - \frac{3}{2}}^B |j = \frac{3}{2}, m_j = \frac{3}{2}\rangle + C_{m, m - \frac{1}{2}, \frac{1}{2}}^{J, L, \frac{3}{2}} |lw\rangle_{m - \frac{1}{2}}^B |j = \frac{3}{2}, m_j = \frac{1}{2}\rangle \\ & + C_{m, m + \frac{1}{2}, -\frac{1}{2}}^{J, L, \frac{3}{2}} |lw\rangle_{m + \frac{1}{2}}^B |j = \frac{3}{2}, m_j = -\frac{1}{2}\rangle + C_{m, m + \frac{3}{2}, -\frac{3}{2}}^{J, L, \frac{3}{2}} |lw\rangle_{m + \frac{3}{2}}^B |j = \frac{3}{2}, m_j = -\frac{3}{2}\rangle \end{aligned} \quad (20)$$

The $C_{m, m_L, m_j}^{J, L, j}$ symbols represent Clebsch-Gordan coefficients. where

$$S_n^0 |lw\rangle^{BF} = \Lambda_n^0 |lw\rangle^{BF} \quad (21)$$

$$\Lambda_n^0 = c_s^{2n} (\nu_s + \frac{1}{2}) \frac{1}{2} + c_d^{2n} (\nu_d + \frac{5}{2}) \frac{1}{2} \quad (22)$$

The eigenvalues of Hamiltonians (11), (12) can then be expressed;

$$E^{(k)} = h^{(k)} + \alpha \Lambda_1^0 + \beta \nu_d (\nu_d + 3) + \delta L(L+1) + \gamma J(J+1) \quad (23)$$

$$E^{(k)} = h^{(k)} + \alpha \Lambda_1^0 + \beta (\nu_1 (\nu_1 + 3) + \nu_2 (\nu_2 + 1)) + \gamma J(J+1) \quad (24)$$

$$h^{(k)} = \sum_{i=1}^k \frac{\alpha}{x_i} \quad (25)$$

The quantum number (k) is related to the total boson number N by

$$N = 2k + \nu_s + \nu_d$$

In order to obtain the numerical results for energy spectra ($E^{(k)}$) of considered nuclei, a set of non-linear *Bethe – Ansatz* equations (BAE) with k- unknowns for k-pair excitations must be solved [17, 18] also constants of Hamiltonian with least square fitting processes to experimental data is obtained. To this aim, we have changed variables as

$$C = \frac{c_s}{c_d} \leq 1, g = 1, y_i = c_d^2 x_i$$

So, the new form of Eq.(16) would be

$$\frac{\alpha}{y_i} = \frac{C^2(\nu_s + \frac{1}{2})}{1 - C^2 y_i} + \frac{(\nu_d + \frac{5}{2})}{1 - y_i} - \sum_{j \neq i} \frac{2}{y_i - y_j} \quad (26)$$

To calculate the roots of *Bethe – Ansatz* equations (BAE) with specified values of ν_s and ν_d , we have solved Eq. (24) with definite values of C and α [16]. Then, we carry out this procedure with different values of C and α to give energy spectra with minimum variation in compare to experimental values [21];

$$\sigma = \left(\frac{1}{N_{tot}} \sum_{i,tot} |E_{exp}(i) - E_{Cal}(i)|^2 \right)^{\frac{1}{2}}$$

(N_{tot} the number of energy levels where included in the fitting processes). The method for optimizing the set of parameters in the Hamiltonian (β, γ, δ) includes carrying out a least-square fit (LSF) of the excitation energies of selected states [16]

III. RESULTS:

This section presented the calculated phase transition observables such as level crossing, ground - state energy and the derivative of the energy, expectation values of the d-boson number operator and energy differences.

A. energy spectrum and level crossing

To display how the energy levels change as a function of the control parameter C and the total number of bosons N, the lowest energy levels as a function of C for $N = 10, 20$ bosons are shown in Fig.3 and Fig.4, where in Fig.3 other fixed parameters are $\alpha = 1000, \beta = -57, \delta = 41, \gamma = 36$ and Fig.4 obtained with $\alpha = 1000, \beta = 6.5, \gamma = -22$. Figs show how the energy levels as a function of the control parameter C evolve from one dynamical symmetry limit to the other. It can be seen from Figs that numerous level crossings occur; especially in the region around $C \geq 0.7$. The crossings are due to the fact that ν_d , $O(5)$ quantum number called seniority, is preserved along the whole path between $O(6)$ and $U(5)$ [22]. With increasing N, level crossing increase that in Fig.4 clearly shows.

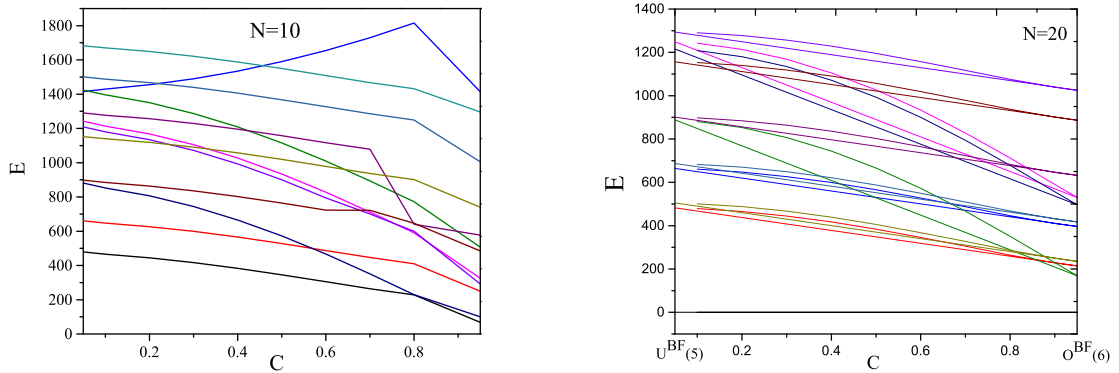


FIG. 3. Energy levels as a function of the control parameter C in the Hamiltonian (11) for $N = 10, 20$ bosons with $\alpha = 1000, \beta = -57, \delta = 41, \gamma = 36$.

B. Ground state energy

The ground state energy is an important observable of phase transition. So, we calculated the ground - state energy $E_{g.s.}$ and its first derivative, $\frac{\partial E_{g.s.}}{\partial C}$. Fig.5 shows changing of the ground - state energy and its first derivative versus

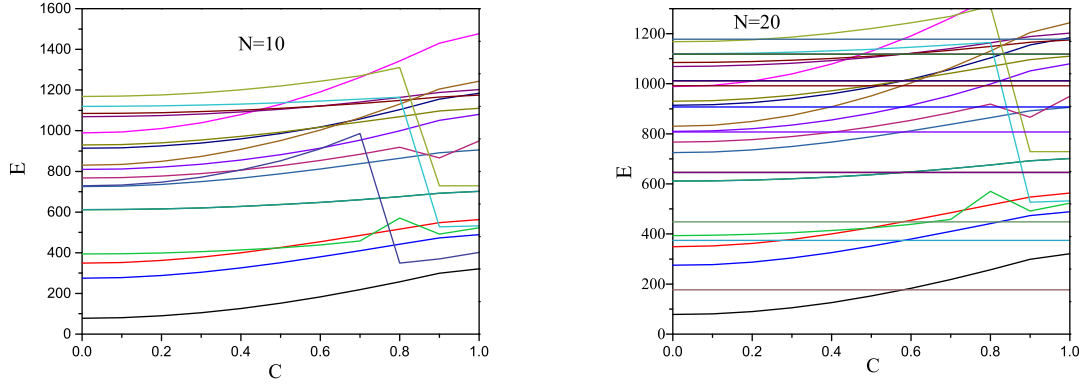


FIG. 4. Same as Fig.3 but with Hamiltonian (12) with $\alpha = 1000, \beta = 6.5, \gamma = -22$.

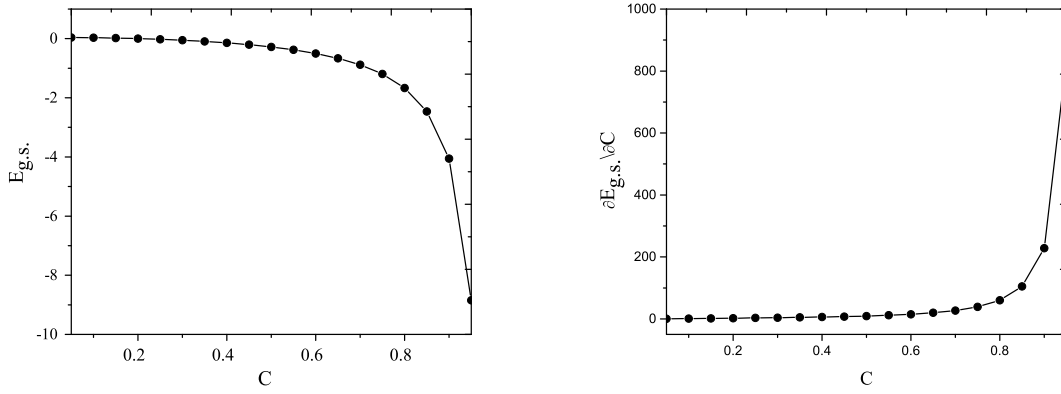


FIG. 5. The ground-state energy (left panel) and derivative of the ground-state energy (right panel) are presented as a function of the control parameter C for a system with 10 bosons.

the control parameter C . Both operators, $E_{g.s.}$ and $\frac{\partial E_{g.s.}}{\partial C}$, are approximately zero in one phase and different from zero in the other phase. Since low N bosons is chosen, it is not possible to distinguish whether the transition is first or second order such as done in the even-even case [23, 24].

C. expectation values of the d-boson number operator

An appropriate quantal order parameter is:

$$\langle \hat{n}_d \rangle = \frac{\langle \psi | \hat{n}_d | \psi \rangle}{N}$$

In order to obtain $\langle \hat{n}_d \rangle$, we act s_m^0 on the eigenstate, $|k; \nu_s \nu n_\Delta LM\rangle$

$$\langle \hat{n}_d \rangle = \frac{2\Lambda_1^0 - 2C^2\Lambda_0^0 + 2k(y_1^{-1} - C)}{1 - C^2} - \frac{5}{2N} \quad (27)$$

Fig.6 shows the expectation values of the d-boson number operator for the lowest states as a function of C control parameter for $N=10$ bosons. Fig.6 displays that the expectation values of the number of d bosons for each J , n_d , remain approximately constant for $C < 0.45$ and only begin to change rapidly for $C > 0.45$. The near constancy of

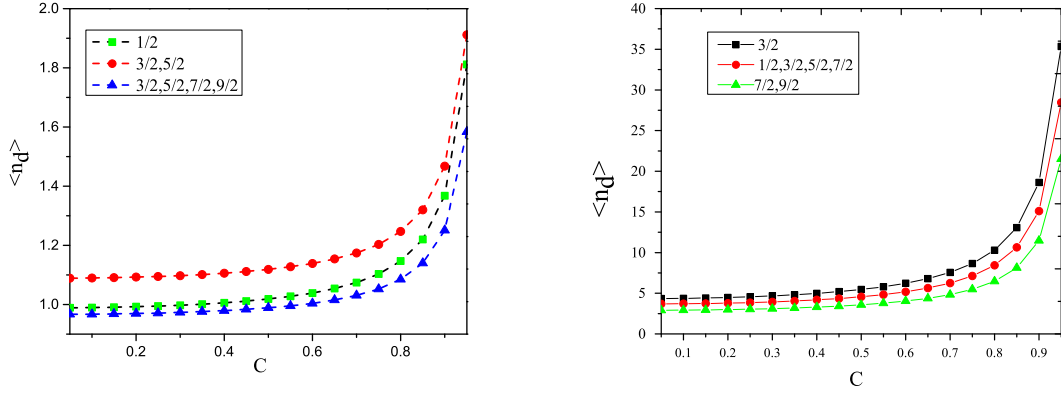


FIG. 6. the expectation values of the d -boson number operator for the lowest states as a function of C control parameter for a $j = 1/2$ particle coupled to a system of (s, d) bosons undergoing a $U(5) - O(6)$ transition (left panel) and single fermion with $j = 3/2$ coupled to a system of (s, d) bosons (right panel).

n_d for $C < 0.45$, is a obvious indication that $U(5)$ dynamical symmetry preserves in this region to a high degree and also the n_d values change rapidly with C over the range $0.65 \leq C \leq 1$. It can be seen from Fig.6 that in due to the presence of the fermion, the transition is made sharper for states that a $j = 1/2$ particle coupled to a system of (s, d) bosons undergoing a $U(5) - O(6)$ transition (left panel) while is made smoother for a $j = 3/2$ particle coupled to a system of (s, d) bosons (right panel).

D. energy differences

Fig.7 displays continues energy differences in terms of control parameter, C , for states that a $j = 1/2$ and $j = 3/2$ particle coupled to a system of (s, d) bosons. Fig.7 shows that during transition from one limit to another exist the points that energy is minimum or maximum near the critical point.

IV. EXPERIMENTAL EVIDENCE

This section presented the calculated results of low-lying states of the odd -A Ba and Rh isotopes. The results include energy levels and the $B(E2)$ values and two-neutron separation energies.

A. energy spectrum

Nuclei in the mass regions around $A \sim 100$ [25, 26] and 130 [27] have transitional characteristics intermediate between spherical and gamma-unstable shapes. The theoretical and experimental studies of energy spectra done in refs.[26, 28, 29] show Rh and Ba isotopes have $U(5) \leftrightarrow O(6)$ transitional characteristics. The possible occurrence of this symmetry in $^{135}_{56}\text{Ba}$ has been recently suggested [28]. The negative parity states in the odd - even nuclei Rh are built mainly on the $2p_{1/2}$ shell model orbit [26]. The single - particle orbits $1g_{7/2}, 2d_{5/2}, 2d_{3/2}$ and $3s_{1/2}$ establish the positive parity states in odd-mass Ba isotopes [29]. In this study, a simplifying assumption is made that single particle states are built on the $j = 1/2$ and $j = 3/2$. We therefore analyze the negative parity states of the odd-proton nuclei, $^{101-109}_{45}\text{Rh}$ and positive parity states of the odd-neutron nuclei, $^{127-137}_{56}\text{Ba}$. In order to obtain energy spectrum and realistic calculation for these nuclei, we need to specify Hamiltonian parameters (11), (12). Eigenvalues of these systems are obtained by solving Bethe-Ansatz equations with least square fitting processes to experimental data to obtain constants of Hamiltonian. The best fits for Hamiltonian's parameters, namely α , β , δ and γ , used in the present work are shown in table 1, 2. Tables 3(a,b...e) and 4(a,b...f) are shown calculated energy spectra along with the experimental values. Figure.8 and Figure.9 are also shown a comparison between the available experimental levels and the predictions of our results for the $^{101-109}_{45}\text{Rh}$ and $^{127-137}_{56}\text{Ba}$ isotopes in the low-lying region of spectra. An acceptable degree

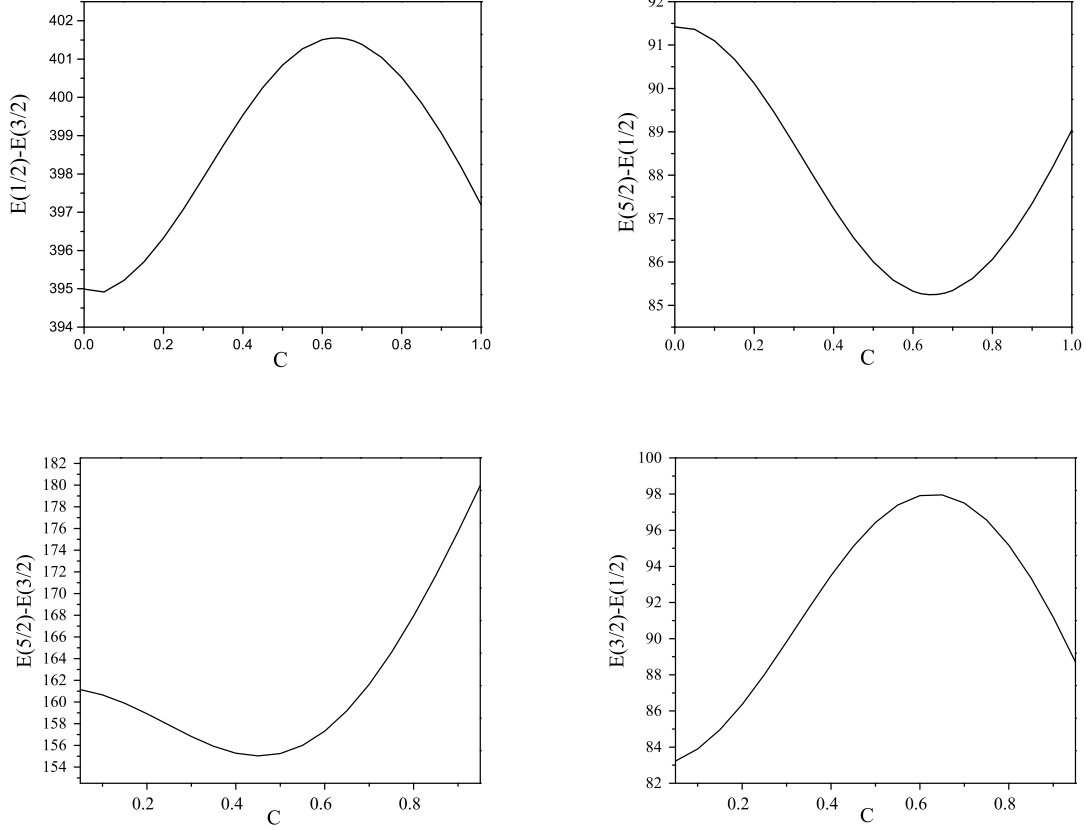


FIG. 7. Continues energy differences in terms of control parameter, C , for states that a $j = 3/2$ (top) and $j = 1/2$ (bottom) particle coupled to a system of (s, d) bosons.

Table 1. Parameters of Hamiltonian (11) used in the calculation of the Rh Isotopes. All parameters are given in keV.

Nucleus	N	C	α	β	δ	γ	σ
$^{101}_{45}Rh$	5	0.06	105.99	4.2572	1.6782	1.2336	194.78
$^{103}_{45}Rh$	6	0.46	52	-1.053	1.5518	23.217	136.5
$^{105}_{45}Rh$	7	0.54	70.61	1.6273	8.7337	-0.0331	97.38
$^{107}_{45}Rh$	8	0.65	196.30	3.5077	-11.734	7.7931	120.48
$^{109}_{45}Rh$	9	0.7	245.269	2.438	-24.072	-181.02	184.79

of agreement is obvious between them. We have tried to extract the best set of parameters which reproduce these complete spectra with minimum variations. It means that our suggestion to use this transitional Hamiltonian for the description of the Rh and Ba isotopic chain would not have any contradiction with other theoretical studies done with special hypotheses about mixing of intruder and normal configurations. On the other hand, predictions of our model for the control parameter of considered nuclei, C , describe the vibrational, i.e. $C = 0$, or rotational, namely $C = 1$, confirm this mixing of both vibrating and rotating structures in these nuclei when $C \sim 0.5 \rightarrow 0.65$. Fig.10 and Fig.11 display a comparison between the calculated continues energy differences and experimental data for Ba and Rh Isotopes, respectively. It can be seen from figs that our results for Ba Isotopes are better than for Rh Isotopes.

One of the most basic structural predictions of $U^{BF}(5) - O^{BF}(6)$ transition is a $\frac{E(\nu_d=2)}{E(\nu_d=1)}$ value. The ratio equal to 2.2 – 2.3 indicates the spectrum of transitional nuclei [3, 4, 15, 30]. Thus we calculated this quantity for Rh and Ba Isotopes. Fig.12 shows $\frac{E(\nu_d=2)}{E(\nu_d=1)}$ prediction values for Rh and Ba Isotopes. For Rh isotopes this value evolves from 2.1 to 2.8 while Ba isotopes vary of 2.85 to 1.9. Fig.12 displays that $\frac{E(\nu_d=2)}{E(\nu_d=1)}$ values for $^{105}_{45}Rh$ and $^{133-135}_{56}Ba$ isotopes are approximately 2.2 – 2.4.

Table 2. Parameters of Hamiltonian (11) and (12) used in the calculation of the Ba Isotopes. All parameters are given in keV.

Nucleus	N	C	α	β	δ	γ	σ
$^{127}_{56}Ba$	8	0.78	3.46	-0.0303	-1.1481	19.098	62.48
$^{129}_{56}Ba$	7	0.8	13.77	-2.685	-8.5776	15.69	94.25
$^{131}_{56}Ba$	6	0.77	0.578	-3.3839	-3.3319	27.12	128
$^{133}_{56}Ba$	5	0.68	21.37	-0.0274	16.46	-1.75	119
$^{135}_{56}Ba$	4	0.65	48.69	3.32	-	36.57	86.84
$^{137}_{56}Ba$	3	0.75	366	0.194	-	23.047	128.6

Table 3a. .Energy spectra for $^{101}_{45}Rh$ isotope

$^{101}_{45}Rh$	J^π	K	ν_d	E_{exp}	E_{cal}
	$(1/2)_1^-$	2	0	0	0
	$(3/2)_1^-$	2	1	305.5	267.3
	$(5/2)_1^-$	2	1	305.5	273.5
	$(3/2)_2^-$	1	2	355.3	665.8
	$(5/2)_2^-$	1	2	355.3	671.9
	$(7/2)_1^-$	1	2	851.4	704.1
	$(9/2)_1^-$	1	2	851.4	715.2
	$(9/2)_2^-$	1	2	899.3	821.2
	$(5/2)_3^-$	2	1	996.4	794.6
	$(3/2)_4^-$	1	2	1058	997.9
	$(5/2)_4^-$	1	2	1058	971.8
	$(1/2)_2^-$	2	0	1531	1472.9

Table 3b. .Energy spectra for $^{103}_{45}Rh$ isotope

$^{103}_{45}Rh$	J^π	K	ν_d	E_{exp}	E_{cal}
	$(1/2)_1^-$	3	0	0	0
	$(3/2)_1^-$	2	1	294.984	334
	$(5/2)_1^-$	2	2	357.408	450.6
	$(1/2)_2^-$	3	0	803.07	598.6
	$(3/2)_2^-$	2	2	803.07	692.4
	$(7/2)_1^-$	2	2	847.58	705.4
	$(5/2)_2^-$	2	2	880.47	809
	$(9/2)_1^-$	2	2	920.1	915.3
	$(3/2)_3^-$	2	1	1277.04	1226.7
	$(13/2)_1^-$	1	3	1637.64	1576.5
	$(15/2)_1^-$	1	4	2221.2	2036.3
	$(17/2)_1^-$	1	4	2345.35	2432.6
	$(17/2)_2^-$	0	5	2418.6	2065.5

Table 3c. .Energy spectra for $^{105}_{45}Rh$ isotope

$^{105}_{45}Rh$	J^π	K	ν_d	E_{exp}	E_{cal}
	$(1/2)_1^-$	3	0	129.781	129.8
	$(3/2)_1^-$	3	1	392.65	229.2
	$(5/2)_1^-$	3	1	455.61	511.7
	$(3/2)_2^-$	2	2	762.11	811.2
	$(3/2)_3^-$	3	1	783	703.6
	$(5/2)_2^-$	2	2	817	821.7
	$(7/2)_1^-$	2	2	817	833.1
	$(5/2)_3^-$	2	3	866	868.2
	$(7/2)_2^-$	2	3	898	861.5
	$(7/2)_3^-$	2	3	976	937.9
	$(9/2)_1^-$	2	2	976	1043.5
	$(3/2)_4^-$	2	2	1147	1312.7
	$(5/2)_4^-$	2	2	1147	1311.9

Table 3d. Energy spectra for $^{107}_{45}Rh$ isotope

$^{107}_{45}Rh$	J^π	K	ν_d	E_{exp}	E_{cal}
	$(1/2)_1^-$	4	0	268.36	268.4
	$(3/2)_1^-$	3	1	485.66	345.4
	$(5/2)_1^-$	3	1	543.84	410.9
	$(9/2)_1^-$	3	2	559.97	423.4
	$(3/2)_2^-$	3	2	752.55	818.8
	$(5/2)_2^-$	3	2	877.75	870.4
	$(3/2)_3^-$	3	1	974.44	921.9
	$(5/2)_3^-$	2	3	974.44	958.6
	$(7/2)_1^-$	3	2	974.44	780.9
	$(3/2)_4^-$	3	2	1009.76	1016.3
	$(5/2)_4^-$	3	2	1009.76	1055.3
	$(7/2)_2^-$	2	3	1251	1099.5
	$(1/2)_2^-$	4	0	1334	1431.2

Table 3e. Energy spectra for $^{109}_{45}Rh$ isotope

$^{109}_{45}Rh$	J^π	K	ν_d	E_{exp}	E_{cal}
	$(1/2)_1^-$	4	0	374.1	374.1
	$(3/2)_1^-$	4	1	568.2	436.6
	$(5/2)_1^-$	4	1	623.2	401.5
	$(3/2)_2^-$	3	2	704.9	758.1
	$(5/2)_2^-$	3	2	856.1	819.6
	$(5/2)_3^-$	3	3	926.9	827.6
	$(3/2)_3^-$	4	1	1162.3	1378.8
	$(3/2)_4^-$	3	2	1214.3	1003.3
	$(5/2)_4^-$	3	2	1283.9	985.6
	$(1/2)_2^-$	4	0	1631	1876.6
	$(3/2)_5^-$	4	1	1631	1627.4
	$(1/2)_3^-$	4	0	1753	1703
	$(3/2)_6^-$	4	1	1753	1750

B. $B(E2)$ transition

The observables such as electric quadrupole transition probabilities, $B(E2)$, as well as quadrupole moment ratios within the low-lying state provide important information about $QPTs$. In this section we discuss the calculation of $E2$ transition strengths and compare the results with the available experimental. The electric quadrupole transition

Table 4a. Energy spectra for $^{127}_{56}Ba$ isotope

$^{127}_{56}Ba$	J^π	K	ν_d	E_{exp}	E_{cal}
	$(1/2)_1^+$	4	0	0	0
	$(3/2)_1^+$	3	1	56.1	51.4
	$(5/2)_1^+$	3	1	81	146.9
	$(7/2)_1^+$	3	2	195.1	265.7
	$(3/2)_2^+$	3	2	269.5	256.3
	$(5/2)_2^+$	3	2	269.5	237.2
	$(7/2)_1^+$	2	3	324.1	376.3
	$(7/2)_2^+$	2	3	374.9	367.1
	$(9/2)_1^+$	3	2	415.6	526.1
	$(11/2)_1^+$	2	3	668.9	723.8
	$(11/2)_2^+$	2	4	867.9	759.4
	$(13/2)_1^+$	2	3	963.6	972.1
	$(15/2)_1^+$	2	4	1291.2	1259.8
	$(15/2)_2^+$	1	5	1519.6	1425.5
	$(17/2)_1^+$	2	4	1654.4	1584.4

Table 4b. .Energy spectra for $^{129}_{56}\text{Ba}$ isotope

$^{129}_{56}\text{Ba}$	J^π	K	ν_d	E_{exp}	E_{cal}
	$(1/2)_1^+$	3	0	0	0
	$(7/2)_1^+$	2	2	8.42	48.71
	$(3/2)_1^+$	3	1	110.56	185.67
	$(3/2)_2^+$	2	2	253.77	210.76
	$(9/2)_1^+$	2	2	263.1	249.9
	$(1/2)_2^+$	3	0	278.58	331.88
	$(3/2)_3^+$	3	1	278.58	228.15
	$(5/2)_1^+$	3	1	278.58	292.83
	$(5/2)_2^+$	2	2	318.4	290.66
	$(3/2)_4^+$	2	2	457.03	483.53
	$(3/2)_5^+$	3	1	459.29	492.44
	$(5/2)_3^+$	2	3	459.29	548.4
	$(7/2)_2^+$	2	3	467.3	589.6
	$(11/2)_1^+$	2	3	544.7	441.257
	$(13/2)_1^+$	2	3	864.1	918.66

Table 4c. .Energy spectra for $^{131}_{56}\text{Ba}$ isotope

$^{131}_{56}\text{Ba}$	J^π	K	ν_d	E_{exp}	E_{cal}
	$(1/2)_1^+$	3	0	0	0
	$(3/2)_1^+$	2	1	108.08	70.5
	$(3/2)_2^+$	2	2	285.25	359.2
	$(5/2)_1^+$	2	1	316.587	424.7
	$(1/2)_2^+$	3	0	365.16	487.8
	$(3/2)_3^+$	2	1	525.85	577.3
	$(5/2)_2^+$	2	2	525.85	494.8
	$(7/2)_1^+$	2	2	543.11	638.1
	$(3/2)_4^+$	2	1	561.752	643.5
	$(5/2)_3^+$	1	3	561.75	473.3
	$(3/2)_5^+$	0	5	718.78	573.7
	$(5/2)_4^+$	2	1	718.78	778.6
	$(1/2)_2^+$	1	3	719.49	698.1
	$(15/2)_1^+$	1	4	1796.4	1821.6
	$(17/2)_1^+$	1	4	2121.7	2282.6

operator $\hat{T}^{(E2)}$ in odd-A nuclei consists of a bosonic and a fermionic part[8, 25]:

$$\hat{T}^{(E2)} = \hat{T}_B^{(E2)} + \hat{T}_F^{(E2)} \quad (28)$$

With

$$T_{B,\mu}^{(E2)} = q_2[s^+ \times \tilde{d} + d^+ \times \tilde{s}]_\mu^{(2)} + q'_2[d^+ \times \tilde{d}]_\mu^{(2)} = q_B Q_{B,\mu} \quad (29)$$

$$Q_{B,\mu} = [s^+ \times \tilde{d} + d^+ \times \tilde{s}]_\mu^{(2)} + \chi[d^+ \times \tilde{d}]_\mu^{(2)} \quad (30)$$

$$T_F^{(E2)} = q_f \sum_{jj'} Q_{jj'} [a_j^+ \times \tilde{a}_{j'}]^{(2)} \quad (31)$$

Where Q_B and $Q_{jj'}$ are the boson and fermion quadrupole operator and q_B and q_f are the effective boson and fermion charges[8, 25]. In the $j = 1/2$ case, the E2 transitions are completely determined by the bosonic part of the E2 operator. The bosonic part have the specific selection rules, where for former term $\Delta\nu_d = \pm 1$, $|\Delta L| \leq 2$ and for latter $\Delta\nu_d = 0, \pm 2$, $|\Delta L| \leq 0, 4$. The reduced electric quadrupole transition rate between the $J_i \rightarrow J_f$ states is given by [8]

$$B(E2; \alpha_i J_i \rightarrow \alpha_f J_f) = \frac{|\langle \alpha_f J_f || T^{(E2)} || \alpha_i J_i \rangle|^2}{2J_i + 1} \quad (32)$$

Table 4d. Energy spectra for $^{133}_{56}\text{Ba}$ isotope

$^{133}_{56}\text{Ba}$	J^π	K	ν_d	E_{exp}	E_{cal}
	$(1/2)_1^+$	2	0	0	0
	$(3/2)_1^+$	2	1	12.3	64.86
	$(5/2)_1^+$	2	1	291.2	175.93
	$(3/2)_2^+$	1	2	302.4	419.34
	$(1/2)_2^+$	1	2	539.8	443.23
	$(7/2)_1^+$	1	2	577.5	701.22
	$(3/2)_3^+$	1	3	630.6	705.24
	$(5/2)_2^+$	1	2	630.6	720.84
	$(5/2)_3^+$	1	3	676.5	675.45
	$(3/2)_4^+$	0	5	676.5	726.015
	$(5/2)_4^+$	1	3	858.5	696.45
	$(9/2)_1^+$	1	2	883.3	933.86
	$(7/2)_2^+$	1	3	1112.3	938.98

Table 4e. Energy spectra for $^{135}_{56}\text{Ba}$ isotope

$^{135}_{56}\text{Ba}$	J^π	K	ν_d	E_{exp}	E_{cal}
	$(3/2)_1^+$	2	0	0	0
	$(1/2)_1^+$	1	1	220.954	200.66
	$(5/2)_1^+$	1	1	480.52	502.31
	$(3/2)_2^+$	1	1	587.82	696.93
	$(3/2)_3^+$	2	0	855	866.9
	$(1/2)_2^+$	1	2	910.35	985.569
	$(5/2)_2^+$	1	2	979.966	904.815
	$(3/2)_4^+$	1	2	979.969	1145.37

The electric quadrupole moment for a state with spin J is given by [8]

$$Q_J = \sqrt{\frac{16\pi}{5}} \left(\frac{J(2J-1)}{(2J+1)(J+1)(2J+3)} \right)^{\frac{1}{2}} \langle J || T^{(E2)} || J \rangle \quad (33)$$

For evaluating B(E2), We consider eigenstates (14) that the normalization factor obtain as:

$$N = \sqrt{\frac{1}{\prod_{p=1}^k \sum_{i=p}^k \left(\frac{2C^2(k-p+\frac{1}{2})(\nu_s+\frac{1}{2})}{(1-C^2 y_{k+1-p})(1-C^2 y_i)} + \frac{2(k-p+\frac{1}{2})(\nu_d+\frac{5}{2})}{(1-y_{k+1-p})(1-y_i)} \right)}} \quad (34)$$

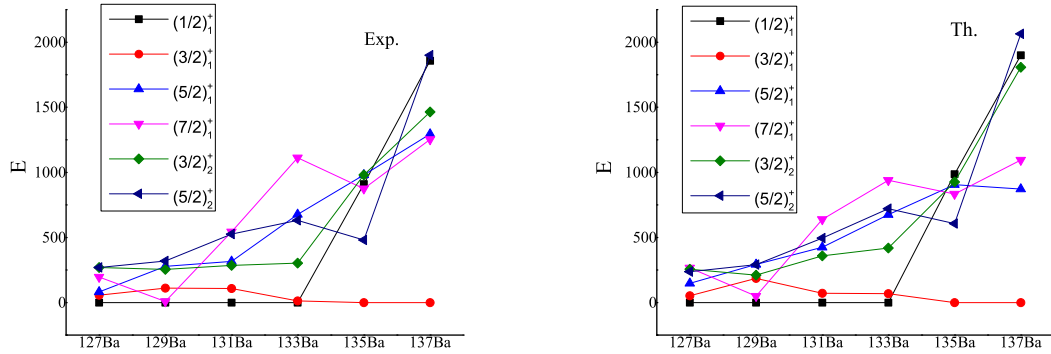


FIG. 8. Comparison between calculated and experimental spectra of positive parity states in Ba Isotopes. The parameters of the calculation are given in Tables 2. In the experimental spectra, taken from [21].

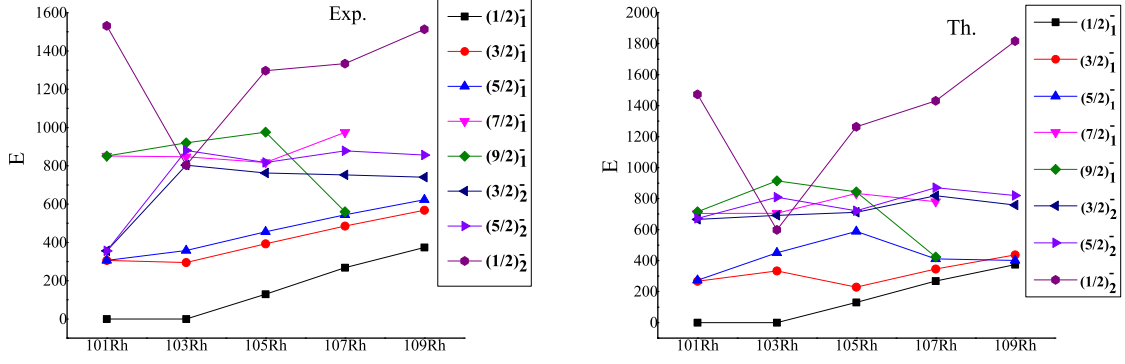


FIG. 9. Comparison between calculated and experimental spectra of negative parity states in Rh Isotopes. The parameters of the calculation are given in Tables 1. In the experimental spectra, taken from [21].

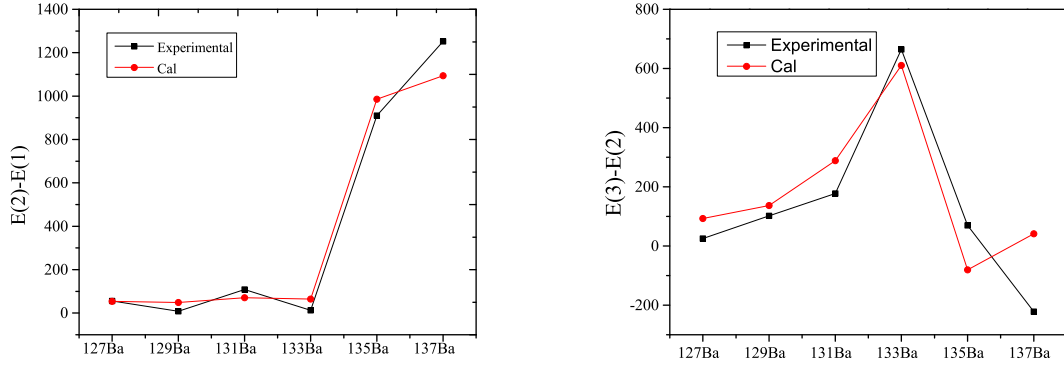


FIG. 10. A comparison between the calculated continues energy differences and experimental data for Ba. In the experimental spectra, taken from [21].

Unfortunately there are very few experimental data available on electromagnetic properties for odd -mass Ba [29] and Rh [25] isotopes. The values of effective charge (q_B, q_f) are tabulated in Table.5. Table.6 shows experimental and calculated values for $B(E2)$ for negative parity states of $^{103}_{45}Rh$, and positive parity states of $^{135}_{56}Ba$. The quadrupole moments for $^{103}_{45}Rh$ and $^{135}_{56}Ba$ also display in table.7. Table.6 and Table.7 show that in general there are a good agreement between the calculated $B(E2)$ values and the quadrupole moments with the experimentally data. The values of the control parameter, C , suggest structural changes in nuclear deformation and shape-phase transitions in Odd-proton Rh isotopes and Odd-neutron Ba isotopes. Because of the effect of single nucleon on the transition and especially critical point, exact selection of the critical point is difficult. With considering this problem, we proposed $C \sim 0.5 - 0.65$ as critical point. So, we conclude from the values of control parameter which has been obtained, $\frac{E(\nu_d=2)}{E(\nu_d=1)}$ value and energy differences that $^{105}_{45}Rh$ and $^{133-135}_{56}Ba$ isotopes are as the best candidates for $U^{BF}(5) - O^{BF}(6)$ transition.

C. two-neutron separation energies

Shape phase transitions in nuclei can be studied experimentally by considering the behavior of the ground state energies of a series of isotopes, or, more conveniently, the behavior of the two-neutron separation energies, S_{2n} [2]. On the other hand, the ground-state two-neutron separation energies, S_{2n} , are observables very sensitive to the

Table 4f. .Energy spectra for $^{137}_{56}Ba$ isotope

$^{137}_{56}Ba$	J^π	K	ν_d	E_{exp}	E_{cal}
	$(3/2)_1^+$	2	0	0	0
	$(7/2)_1^+$	1	1	1252.5	1093.9
	$(5/2)_1^+$	1	1	1294	1124.25
	$(3/2)_2^+$	1	1	1463.8	1607
	$(1/2)_1^+$	1	2	1857	1782.9
	$(1/2)_2^+$	1	1	1836.2	1899.2
	$(3/2)_3^+$	2	0	2041	1940.4
	$(5/2)_2^+$	1	2	1899	2063.2
	$(3/2)_4^+$	1	2	1899	1883.5
	$(5/2)_3^+$	1	2	2041	2041.8
	$(7/2)_2^+$	1	2	2230	2258
	$(9/2)_1^+$	1	2	2230	2248.6
	$(7/2)_3^+$	1	2	2340	2263.5
	$(7/2)_4^+$	1	1	2423.8	2530

Table 5.The coefficients of $T(E_2)$ used in the present work for $^{103}_{45}Rh$ Rh and $^{135}_{56}Ba$ isotopes.

Nucleus	$q_B(eb)$	$q_f(eb)$
$^{103}_{45}Rh$	0.461	0
$^{135}_{56}Ba$	4.7329	-0.7194

Table 6. B(E2)values for $^{103}_{45}Rh$ and $^{135}_{56}Ba$ isotopes. The experimental data for $^{103}_{45}Rh$ isotope are taken from [21, 25].The experimental data for $^{135}_{56}Ba$ isotope are taken from [21, 31].

Nucleus	$J_i^\pi \rightarrow J_j^\pi$	$B(E_2(e^2b^2))$	
		$exp.$	$calc.$
$^{103}_{45}Rh$	$(3/2)_1^- \rightarrow (1/2)_1^-$	0.109	0.1172
	$(5/2)_1^- \rightarrow (1/2)_1^-$	0.118	0.1172
	$(5/2)_2^- \rightarrow (1/2)_1^-$	0.0044	0.0044
	$(5/2)_2^- \rightarrow (3/2)_1^-$	0.0768	0.0645
	$(5/2)_2^- \rightarrow (5/2)_1^-$	0.015	0.0097
	$(7/2)_1^- \rightarrow (3/2)_1^-$	0.13	0.1165
	$(9/2)_1^- \rightarrow (5/2)_1^-$	0.179	0.1349
$^{135}_{56}Ba$	$(1/2)_1^+ \rightarrow (3/2)_1^+$	4.6	3.696
	$(5/2)_1^+ \rightarrow (1/2)_1^+$	2.65	2.913
	$(7/2)_1^+ \rightarrow (3/2)_1^+$	19.9	14.784
	$(1/2)_2^+ \rightarrow (3/2)_1^+$	11.7	14.403
	$(3/2)_2^+ \rightarrow (3/2)_1^+$	18	14.785
	$(3/2)_3^+ \rightarrow (3/2)_1^+$	7	7.001

Table 7. Quadrupole moments for $^{103}_{45}Rh$ and $^{135}_{56}Ba$ isotopes. The experimental data for $^{103}_{45}Rh$ isotope are taken from [21, 25].The experimental data for $^{135}_{56}Ba$ isotope are taken from [21, 31].

Nucleus	J^π	$Q(eb)$	
		$exp.$	$calc.$
$^{103}_{45}Rh$	$(1/2)_1^-$	0	0
	$(3/2)_1^-$	-0.32	-0.2588
	$(5/2)_1^-$	-0.41	-0.2824
	$(7/2)_1^-$	-	-0.5538
	$(9/2)_1^-$	-	-0.5355
$^{135}_{56}Ba$	$(3/2)_1^+$	0.146	0.1509
	$(1/2)_1^+$	-	0.1349
	$(5/2)_1^+$	-	0.5126

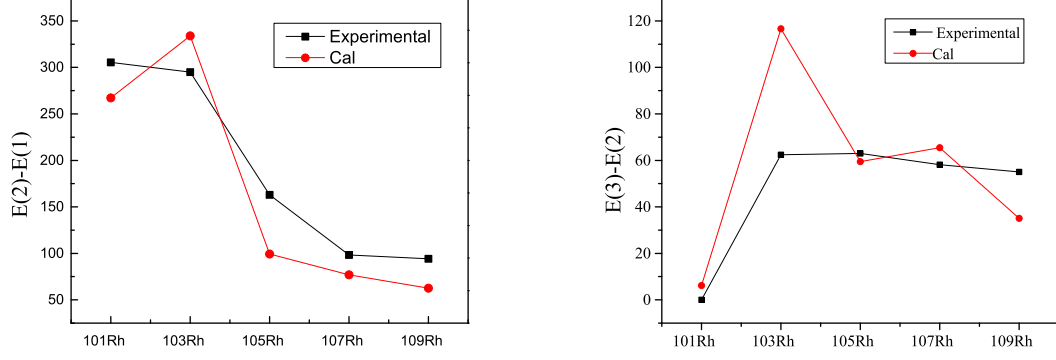


FIG. 11. A comparison between the calculated continues energy differences and experimental data for Rh. In the experimental spectra, taken from [21].

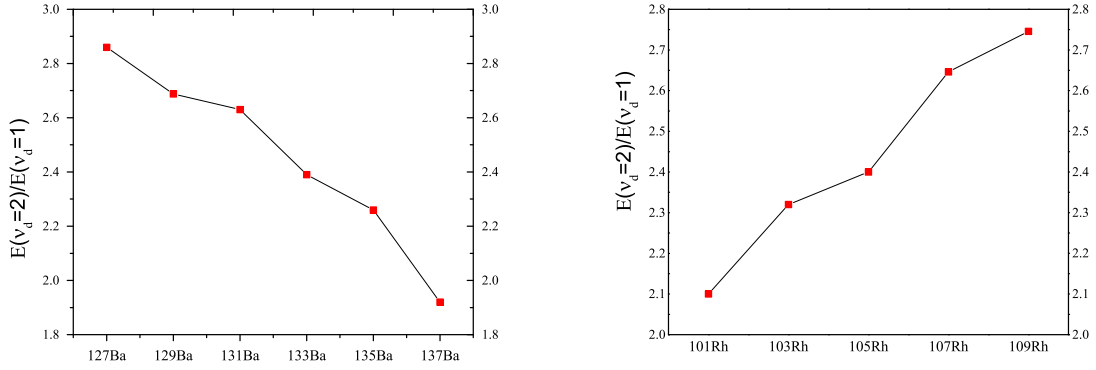


FIG. 12. $(E(\nu_d = 2))/(E(\nu_d = 1))$ prediction values for Rh and Ba Isotopes. In the experimental spectra, taken from [21]

details of the nuclear structure. The occurrence of continuities in the behavior of two-neutron separation energies describe a second-order shape-phase transition between spherical and γ - *unstable* rotor limits [2, 32]. In due to, we have investigated the evolution of two-neutron separation energies along the Ba and Rh isotopic chains by both experimental and theoretical values, which have been presented in Fig.13. The binding energy as a function of proton and neutron number is given by [2]

$$E_B(N_\pi, N_\nu) = E_c + A_\pi N_\pi + A_\nu N_\nu + \frac{1}{2} B_\pi N_\pi (N_\pi - 1) + \frac{1}{2} B_\nu N_\nu (N_\nu - 1) + C N_\pi N_\nu + E_D(N_\pi, N_\nu) \quad (35)$$

Where $N_\pi(N_\nu)$ is the number of proton (neutron) bosons in the valence shell, E_c the contribution from the core and E_D is the contribution to the binding energy due to the deformation. Using Eq. (35), one obtains the following relation for the two-neutron separation energy[2]:

$$S_{2n}(N_\pi, N_\nu) = E_B(N_\pi, N_\nu) - E_B(N_\pi, N_\nu - 1) = A_n + B N_\pi + C_n N_\nu + (E_D(N_\pi, N_\nu) - E_D(N_\pi, N_\nu - 1)) \quad (36)$$

Using the S_{2n} empirical values for these isotopic chains [21] we have extracted $A_n + B N_\pi = 15.1857, 20.526$ Mev and $C_n = 1.9784, -1.7992$ Mev for Ba and Rh, respectively. Then, we obtained the two-neutron separation energies, which are shown in Fig.13, together with the experimental values. It can be seen from Fig.13 that exist continuities (linear variation) in the behavior of two-neutron separation energies thus the phase transition for Ba and Rh isotopic chains is of second order. Our result Confirmed the predictions of done in refs. [2, 32], where they suggest a linear variation of S_{2n} with respect to the neutron number for the $U(5) - SO(6)$ transitional region.

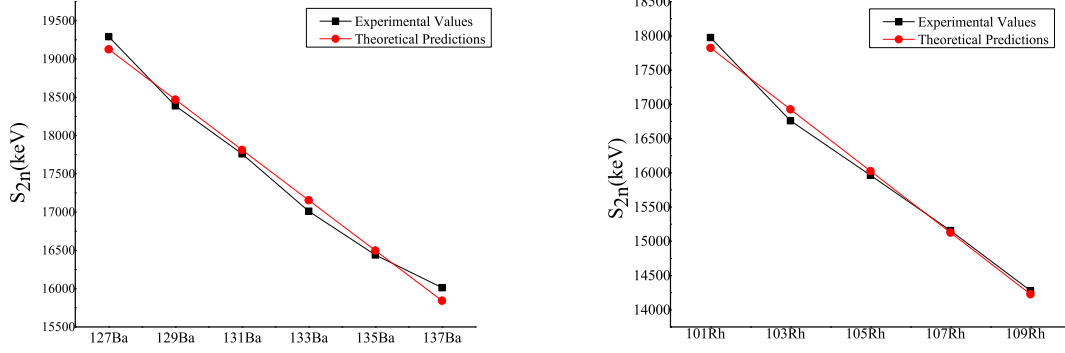


FIG. 13. A comparison between theoretical and experimental two neutron separation energies, S_{2n} (in keV) for Ba isotopes (left panel) and Rh isotopes (right panel). Experimental data from [21].

V. CONCLUSIONS

In this paper, we have analyzed transition from spherical to γ -unstable shapes in odd- A nuclei. Key observables of phase transition such as level crossing, ground-state energy and derivative of the ground-state energy and expectation values of the d-boson number operator have calculated. We have presented experimental evidence for the $U(5) - O(6)$ transition for negative parity states of the $^{101-109}_{45}\text{Rh}$ isotopic chain and positive parity states of $^{127-137}_{56}\text{Ba}$ isotopic chain, and performed an analysis for these isotopes via a $SU(1,1)$ -based Hamiltonian. The results indicate that the energy spectra of the Rh and Ba isotopes can be reproduced quite well. The calculated $B(E2)$ values and two neutron separation energies are in agreement with the available experimental data. Our results show that Rh isotopes have gamma-unstable rotor features but the vibrational character is dominant while a dominance of dynamical symmetry $O(6)$ exist for Ba isotopic chain and also $^{105}_{45}\text{Rh}$ and $^{133-135}_{56}\text{Ba}$ isotopes are as the best candidates for $U^{BF}(5) - O^{BF}(6)$ transition.

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